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Solution by G. I. HOPKINS, J. SCHEFFER, and G. B. M. ZERR.

Let AB be the sum of the two sides, CD the bisector, and F the given angle. Make $\angle HGK = \angle F$ and bisector $GL = CD$. Draw LM parallel to HG and NL parallel to GK . Extend GK making $GO = NG$. Make $OP = AB$, and on MP draw the semicircle MSP . Draw the perpendicular $MQ = ML$, also QR parallel to MP and ST perpendicular to MP . Through L draw TX , then TGX is the required triangle; for $TS^2 = MT \cdot PT = QM^2 = ML^2$. From similar triangles XNL and LMT , $XN:LM::NL:MT$. Since $MLNG$ is a rhombus $LM = NL$. $\therefore XN \cdot MT = ML^2$. $\therefore MT \cdot TP = MT \cdot XN$, whence $NX = TP$. $\therefore GX + GP = OP = AB$.

CALCULUS.

168. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

The tangent of what Cartesian curve makes an x -intercept always m times as long as the corresponding y -intercept.

Solution by J. SCHEFFER.

Let the equation of the tangent be $y - y' = \frac{dy'}{dx}(x - x')$. Consequently the x -intercept $= x' - y' \frac{dx'}{dy'}$, and the y -intercept $= y' - x' \frac{dy'}{dx}$; therefore, omitting the accents, by the condition imposed upon the problem

$$-\frac{y}{p} + x = m(y - px); \quad \left(p = \frac{dy}{dx}\right),$$

whence $m x p^2 - (m y - x)p - y = 0$; or, arranged differently, $(px - y)(mp + 1) = 0$, whence $px - y = 0$ and $mp + 1 = 0$. From the former of these two equations we get $y = ax$; and from the second $my + x = b$, where a and b are arbitrary constants. Both equations represent straight lines, the first one of which passes through the origin.

MECHANICS.

159. Proposed by J. E. SANDERS, Hackney, Ohio.

Required the time for a tree, considered as a material line of uniform density, length $a = 100$ feet, to fall; the tree being inclined $\phi = 1^\circ$ from perpendicular.

Solution by the PROPOSER.

From Mechanics we find that

$$\frac{\sqrt{(2gh)t}}{l} = \int \frac{d\theta}{\sqrt{[1 - (2l/h)\sin^2 \frac{1}{2}\theta]}} \dots (1),$$

where g =gravity=32, l =length to center of oscillation= $\frac{2}{3}a$, h =height of zero velocity= $2l$ without a sensible error, t =time, θ =angle counted from the down point. Reducing, we have,

$$t = \frac{1}{2} \sqrt{\left(\frac{l}{g}\right)} \int_{\frac{1}{2}\pi}^{\pi-\phi} \frac{d\theta}{\cos \frac{1}{2}\theta} = \sqrt{\left(\frac{l}{g}\right)} \left[\log \tan\left(\frac{1}{4}\pi + \frac{1}{4}\theta\right) \right]_{\frac{1}{2}\pi}^{\pi-\phi}$$

$$= \sqrt{\left(\frac{l}{g}\right)} \log \frac{\cot \frac{1}{4}\phi}{\tan \frac{3}{8}\pi} = \sqrt{\left(\frac{l}{g}\right)} (8.647491).$$

But $l = \frac{2}{3}$ of 100. $\therefore t = \sqrt{\left(\frac{100}{32}\right)} (8.647491) = 12.481$ seconds.

AVERAGE AND PROBABILITY.

139. Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.

Four points are taken at random on the surface of a given sphere; find the average volume of the tetrahedron formed by the planes passing through the points taken three and three.

No solution has been received.

140. Proposed by L. C. WALKER, A.M., Professor of Mathematics, Colorado School of Mines, Golden, Col.

Obtain the average area of a triangle formed by a tangent to the four-cusped hypocycloid and the co-ordinate axes.

Solution by J. E. SANDERS, Hackney, Ohio.

The curve's equation is $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ The length of the perpendicular from the origin on the tangent is

$$\frac{xdx - ydy}{\sqrt{(dx^2 + dy^2)}} = \frac{x^{\frac{2}{3}} + y^{\frac{2}{3}}}{\sqrt{(y^{-\frac{2}{3}} + x^{-\frac{2}{3}})}} = \frac{2}{3} \sqrt{(axy)}.$$

The equation to the tangent is $x'/x^{\frac{1}{3}} + y'/y^{\frac{1}{3}} = a^{\frac{1}{3}}$, and its length between the axis= $a^{\frac{2}{3}} \sqrt{(x^{\frac{2}{3}} + y^{\frac{2}{3}})} = a$.

$\therefore$ The area of the triangle is $A = \frac{1}{2} a^{\frac{2}{3}} \sqrt{(axy)} = \frac{1}{2} a^2 \sin \theta \cos \theta$, if $x = a \cos^3 \theta$, and $y = a \sin^3 \theta$.

Then, if the tangent is through any point on the curve, the average area is

$$\Delta = \frac{1}{s} \int A ds.$$

But $ds = a^{\frac{1}{3}} x^{-\frac{1}{3}} dx = 3a \sin \theta \cos \theta d\theta$ and $s = \frac{2}{3} a$.

$\therefore \Delta = a^2 \int_0^{\frac{1}{2}\pi} \sin^2 \theta \cos^2 \theta d\theta = \frac{1}{16} \pi a^2$, or, if the tangent is to be a random line, the average area is